Reliability, Confidence, and Sample Size, Part II

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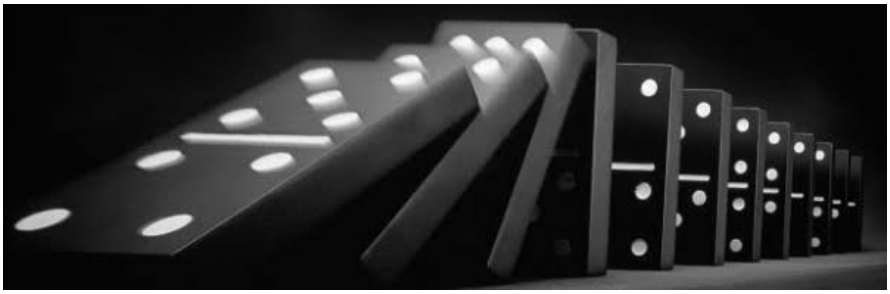


In Part I the problem was to find the sample size, n, given failure count, c = 0, confidence level = 1 – P(c = 0), and minimum reliability = (1 – p'). The table giving sample size, n, with failures c = 0, for certain common combinations of confidence level and minimum reliability is reproduced below.

Minimum Reliability (1 - p')	Confidence Level (1 - P(0))			
	0.90	0.95	0.98	0.99
0.998	1150	1496	1954	2300
0.995	459	598	780	919
0.990	229	298	389	458
0.950	45	58	76	90
0.900	22	28	37	44

While I would like that none of the samples fail testing, failures do happen. Does that mean testing should stop on first fail? Are the test results useless? In this part I will flip the script. I will talk about what value I can extract from test results if I

encounter one or more failures in the test sample.

I start with the binomial formula as before

$$P(x=c)=C\binom{n}{x}(p')^{x}(q')^{n-x}$$

It gives us the likelihood, P(x = c), of finding exactly c failures in n samples for a particular population failure rate p'. (Note that 1 – P(x ≤ c) is our confidence level, and 1 – p' = q' is our desired reliability.)

However, knowing the likelihood of finding just c failures in n samples isn't enough. Different samples of size n from the same population will give different counts of failures c. If I am okay with c failures in n samples, then I must be okay with less than c failures, too! Therefore, I need to know the cumulative likelihood of finding c or less failures in n samples, or P(x ≤ c). That likelihood is calculated as the sum of the individual probabilities. For example, if c = 2 samples fail, I calculate P(x ≤ 2) = P(x = 2) + P(x = 1) + P(x = 0).

For a particular failure rate p', I can make the statement that my confidence is 1 – P(x ≤ c) that the failure rate is no greater than p' or alternatively my reliability is no less than q' = (1 – p').

It is useful to build a plot of P(x ≤ c) versus p' to understand the relationship between the two for a given sample size n and failure count c. This plot is referred to as the operating characteristic (OC) curve for a particular n and c combination.

For example, given n = 45, and c = 2, my calculations would look like:

$$P(x=2)=C\binom{45}{2}(p')^{2}(q')^{45-2}$$

$$P(x=1)=C\binom{45}{1}(p')^{1}(q')^{45-1}$$

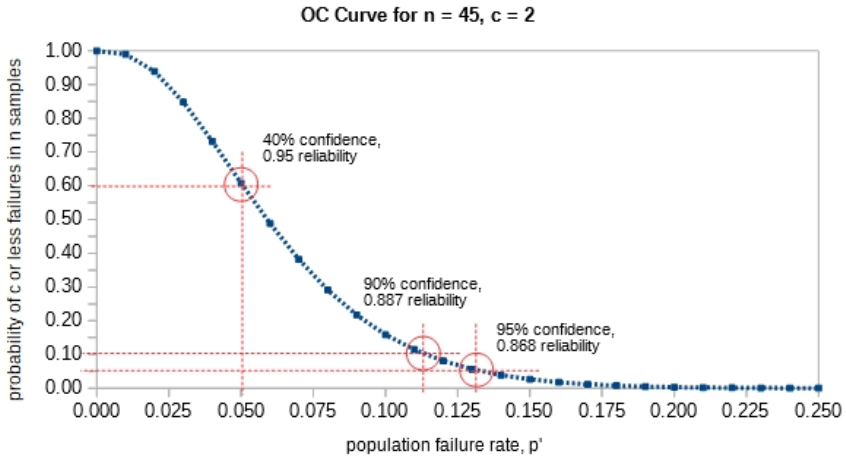
$$P(x=0)=C\binom{45}{0}(p')^{0}(q')^{45-0}$$

$$P(x\leq 2)=P(x=2)+P(x=1)+P(x=0)$$

The table below shows a few values that were calculated:

p'	P(x = 2)	P(x = 1)	P(x = 0)	P(c ≤ 2)
0.00	0.0000	0.0000	1.0000	1.0000
0.02	0.1661	0.3700	0.4029	0.9390
0.04	0.2738	0.2987	0.1593	0.7318
0.06	0.2491	0.1774	0.0618	0.4883
...	...	...	...	...

A plot of P(c ≤ 2) versus p' looks like:



From the plot I can see that the more confidence I require, the higher the failure rate or lesser the reliability estimate will be (e.g. 90% confidence with 0.887 reliability, or 95% confidence with 0.868 reliability.) Viewed differently, the more reliability I require, the less confidence I have in my estimate (e.g. 0.95 reliability with 40% confidence level).

Which combination of confidence and reliability to use depends on the user's needs. There is no prescription for choosing one over another.

I may have chosen a sample size of n = 45 expecting c = 0 failures for testing with the expectation of having 90% confidence at 0.95 reliability in my results. But just because I got c = 2 failures doesn't mean the effort is for naught. I could plot the OC curve for the combination of n, and c to understand how my confidence and reliability has been affected. Maybe there is a combination that is acceptable. Of course, I would need to explain why the new confidence, and reliability levels are acceptable if I started with something else.

Operating characteristic curves can be constructed in MS Excel or Libre Calc with

the help of BINOM.DIST(c, n, p’, 1) function.

	A	B	C
1	p’	P(c ≤ 2)	P(c ≤ 2)
2	0.00	‘=BINOM.DIST(2,45,A2,1)	1.0000
3	0.02	‘=BINOM.DIST(2,45,A3,1)	0.9390
4	0.04	‘=BINOM.DIST(2,45,A4,1)	0.7318
5	0.06	‘=BINOM.DIST(2,45,A5,1)	0.4883
6	...	...	...

Once I have values for p’ and P(c ≤ 2), I can create an X-Y graph with X = p’, and Y = P(c ≤ 2).

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